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AVERAGE TRANSMISSION LOSS FOR MODEL SOFAR CHANNELS.(U)  
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# INTRODUCTION

Transmission loss in the deep sound channel is usually regarded as arising from spherical spreading out to a transition range  $R_0$ , with cylindrical spreading beyond, plus an attenuation loss proportional to range. The purpose of this memorandum is to examine SOFAR transmission loss using ray theory, the study having been prompted by a great variation in the experimentally determined values of  $R_0$ .<sup>2</sup>

Under the assumption that ray theory is applicable, and that the attenuation is the same along all rays, an expression can be obtained for the average acoustic

\*By average intensity is meant intensity smeared out over any interference structure of the field.

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intensity in the SOFAR channel at long ranges. It is noted that Urlick<sup>3</sup> has presented similar, but incomplete, results for the same problem.

#### AVERAGE INTENSITY AT LONG RANGE

Consider a thin bundle of rays with angular spread  $\Delta\theta_0$ , emerging at an angle  $\theta_0$  from a source located on the SOFAR channel axis. Assuming that the glancing angle of the bundle is  $\theta_1$  at a receiver located at range  $R$  from the source and level  $z_1$ , the intensity at the receiver is given by<sup>4</sup>

$$\Delta I = \frac{I_0 \cos \theta_0 \Delta \theta_0}{R \sin \theta_1 \Delta R} 10^{-(\alpha/10)R}, \quad (1)$$

where  $I_0$  is the source strength,  $\Delta R$  is the horizontal spread of the bundle, and  $\alpha$  is the attenuation coefficient. For simplicity, the following development is for channels symmetric about the axis; the extension to asymmetric channels is straight forward. It is now assumed that, at a sufficiently long range, the energy within a ray bundle becomes distributed over the entire channel width accessible to it. Letting  $X(\theta_0)$  be the range at which the ray reaches its apex, and letting  $N$  be the number of axis crossings of the ray over range  $R$ , it can easily be seen that the energy becomes spread over the track width when

$$(2N+1)X(\theta_0) = (2N-1)X(\theta_0 + \Delta\theta_0). \quad (2)$$

This condition can be rewritten as

$$2X(\theta_0) = \left(1 + \frac{1}{2N}\right) \frac{dR}{d\theta_0} \Delta\theta_0. \quad (3)$$

Letting  $\Delta\theta_0 \rightarrow d\theta_0$ , Equations (1) and (3) yield for the total intensity at  $(R, z_1)$ :

$$I \approx \frac{I_0}{R} \int_{\theta_{0\min}}^{\theta_{0\max}} \frac{\cos \theta_0 d\theta_0}{2 \sin \theta_1 X(\theta_0)} 10^{-(\alpha/10)R}, \quad (4)$$

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where  $\theta_{0 \min}$  is the minimum glancing angle of a ray reaching the level of the receiver ( $z_1$ ), and  $\theta_{0 \max}$  is the largest glancing angle of the channeled rays.

Equation (4) can be put in a more useful form using Snell's law,

$$I = \frac{I_0}{R} \int_{\theta_{0 \min}}^{\theta_{0 \max}} \frac{\cos \theta_0 d\theta_0}{2 \left[ 1 - \frac{c^2(z_1)}{c_0^2} \cos^2 \theta_0 \right]^{1/2}} 10^{-(\alpha/10)R} \chi(\theta_0) \quad (5)$$

In the above  $c_0$  is the sound velocity on the channel axis and  $c(z_1)$  is the sound velocity at level  $z_1$ . Assuming the usual expression for SOFAR propagation loss,

$$I = \frac{I_0}{R R_0} 10^{-(\alpha/10)R} \quad (6)$$

the following association can be made:

$$R_0^{-1} = \int_{\theta_{0 \min}}^{\theta_{0 \max}} \frac{\cos \theta_0 d\theta_0}{2 \left[ 1 - \frac{c^2(z_1)}{c_0^2} \cos^2 \theta_0 \right]^{1/2}} \chi(\theta_0) \quad (7)$$

### Examples

By way of illustration, the above expression for  $R_0$  will be applied to the " $\beta$  family" of model SOFAR channels investigated by Hirsch and Carter.<sup>5</sup> velocity profiles for this class are

$$c(z) = c_0 [1 - \alpha |z|^\beta]^{-1/2} \quad (8)$$

It is shown in Reference (5) that

$$\chi(\theta_0) = \frac{\sin^{2/\beta} \theta_0}{\alpha \beta \tan \theta_0} B\left(\frac{1}{\beta}, \frac{1}{2}\right), \quad (9)$$

where  $B(x, y)$  is Euler's integral of the first kind.



Using Eqs. (7) and (8) the expression for  $R_0$  becomes

$$R_0^{-1} = \frac{2\alpha\beta}{\sqrt{\pi}} \frac{\Gamma(\frac{1}{\beta} + \frac{1}{2})}{\Gamma(\frac{1}{\beta})} [1 - |\alpha z_1|^\beta]^{\frac{1}{2}} \int_0^{\pi/2} \frac{\sin^{1-2/\beta} \theta_0 d\theta_0}{[\sin^2 \theta_0 - |\alpha z_1|^\beta]} \quad (10)$$

$$= \frac{\alpha\beta}{\sqrt{\pi}} \frac{\Gamma(\frac{1}{\beta} + \frac{1}{2})}{\Gamma(\frac{1}{\beta})} [1 - |\alpha z_1|^\beta]^{\frac{1}{2}} \int_{|\alpha z_1|^\beta}^1 \frac{dy}{y^{1/\beta} \sqrt{(1-y)(y - |\alpha z_1|^\beta)}}.$$

For  $\beta = 1$ , Eq. (9) can be integrated<sup>6</sup> and yields

$$R_0^{-1}(\beta=1) = \frac{\sqrt{\pi}\alpha}{2} \left[ \frac{1 - |\alpha z_1|}{|\alpha z_1|} \right]^{\frac{1}{2}}. \quad (11)$$

Similarly the expression when  $\beta = 2$  is<sup>7</sup>

$$R_0^{-1}(\beta=2) = \frac{\alpha}{\pi} [1 - |\alpha z_1|^2]^{\frac{1}{2}} K(\sqrt{1 - |\alpha z_1|^2}), \quad (12)$$

where  $K(m)$  is the complete elliptic integral of the first kind.

It is to be noted that, with  $1 \leq \beta \leq 2$ , the intensity expression diverges when  $z_1 = 0$ , that is, when the receiver is on the channel axis. This is to be expected in the ray approximation.<sup>8</sup>

The case  $\beta > 2$ <sup>\*\*</sup> can be evaluated simply when the receiver is on the axis. Thus,

$$R_0^{-1} = \frac{2\alpha\beta}{\sqrt{\pi}} \frac{\Gamma(\frac{1}{\beta} + \frac{1}{2})}{\Gamma(\frac{1}{\beta})} \int_0^{\pi/2} \sin^{-2/\beta} \theta_0 d\theta_0$$

$$= \frac{\alpha\beta}{4^{1/\beta}} \frac{B(\frac{1}{2} - \frac{1}{\beta}, \frac{1}{2} - \frac{1}{\beta})}{B(\frac{1}{2}, \frac{1}{\beta})}. \quad (13)$$

for  $z = 0, \beta > 2$ .

<sup>\*\*</sup>Carter and Hirsch<sup>5</sup> have demonstrated that unrealistic SOFAR channel models are given by  $\beta \geq 2$ . This case is included only for the sake of illustration.

As a final example a somewhat more practical problem will be considered. Assume the existence of a constant velocity layer of thickness  $h$  in the vicinity of the SOFAR channel axis. Above and below the layer constant velocity gradients ( $c_0 g_1$ ,  $c_0 g_2$ ) extending to the ocean surface and bottom respectively, are assumed. The distance from the top of the layer to the surface is taken to be  $h_1$  and from the bottom of the layer to the ocean floor to be  $h_2$ . For simplicity, assume that all rays reaching the surface and bottom are attenuated. Then, with the receiver on the axis, Eq. (6) gives<sup>9</sup>

$$R_o^{-1} = \int_0^{\Theta} \left[ h + \frac{\tan^2 \theta_o}{g_{\text{eff}}} \right]^{-1} d\theta_o, \quad (14)$$

where

$$\Theta = \min \left\{ \cos^{-1} \frac{1}{1 + g_1 h_1}, \cos^{-1} \frac{1}{1 + g_2 h_2} \right\}, \quad \frac{1}{g_{\text{eff}}} = \frac{1}{g_1} + \frac{1}{g_2}.$$

Performing the integration in Eq. (14) yields

$$R_o^{-1} = \frac{1}{h - \frac{1}{g_{\text{eff}}}} \left[ \Theta - \sqrt{\frac{1}{h g_{\text{eff}}}} \tan^{-1} \sqrt{\frac{1}{h g_{\text{eff}}}} \tan \Theta \right] \quad (15)$$

The following values are taken as being typical of the Western Atlantic:<sup>10</sup>

$$\begin{aligned} h &= 900 \text{ ft} & , & & c_0 &= 4,880 \text{ ft/sec} \\ h_1 &= 3,750 \text{ ft} & , & & c_1 &= 5,000 \text{ ft/sec} \\ h_2 &= 11,350 \text{ ft} & , & & c_2 &= 5,050 \text{ ft/sec} \end{aligned}$$

Substitution of these values into Eq. (15) indicates a value of  $R_0 \sim 5$  kyds (4.6 km). It is to be noted that this figure is between one and two orders of magnitude smaller than most of the experimentally determined values of  $R_0$ . 11

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